

Exercise 78

Repeating Decimal Express each repeating decimal as a fraction. (See the margin note on page 3.)

$$(a) 5.\overline{23} \qquad (b) 1.\overline{37} \qquad (c) 2.\overline{135}$$

Solution**Part (a)**

Let $x = 5.2323\cdots$. Then

$$100x = 523.2323\cdots$$

Subtract x from both sides.

$$100x - x = 523.2323\cdots - x$$

$$99x = 523.2323\cdots - 5.2323\cdots$$

$$99x = 518$$

Solve for x .

$$x = \frac{518}{99}$$

Therefore,

$$5.\overline{23} = \frac{518}{99}.$$

Part (b)

Let $x = 1.3777\cdots$. Then

$$100x = 137.777\cdots$$

$$10x = 13.777\cdots$$

Subtract the respective sides of these equations.

$$90x = 124$$

Solve for x .

$$x = \frac{124}{90} = \frac{62}{45}$$

Therefore,

$$1.\overline{37} = \frac{62}{45}.$$

Part (c)

Let $x = 2.1353535 \dots$. Then

$$10000x = 21353.5353 \dots$$

$$100x = 213.5353 \dots$$

Subtract the respective sides of these equations.

$$9900x = 21140$$

Solve for x .

$$x = \frac{21140}{9900} = \frac{1057}{495}$$

Therefore,

$$2.1\overline{35} = \frac{1057}{495}.$$